A Deontic Logic of Knowingly Complying

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- $\mathsf{U}(i)$ is an indistinguishability relation between plans for agent i.

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Note: $A\varphi = N(\neg\varphi, \bot)$ and $E\varphi = \neg A \neg\varphi$.

An example: Fire Emergency Evacuation Plan

EMERGENCY PROCEDURE

– FIRE – SMOKE – EXPLOSION	KEEP CALM PULL FIRE ALARM, FROM A SAFE LOCATION CALL 999 (FIRE BRIGADE)

Evacuate: close doors behind, use only stairs or ramps.

If unsafe to evacuate: shut door, block cracks, stay low near window.

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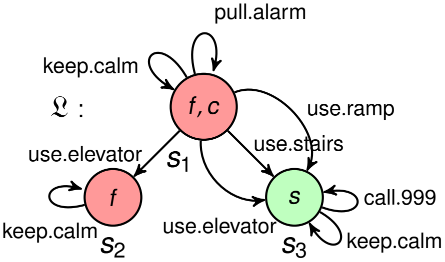

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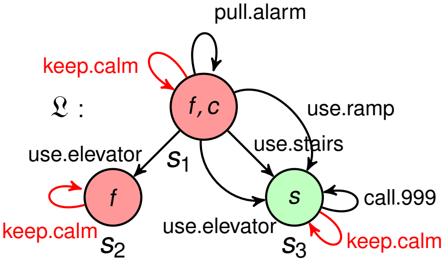

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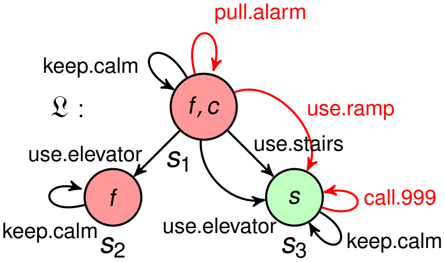

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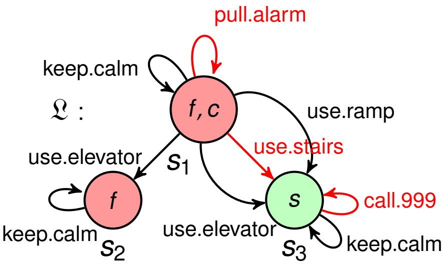

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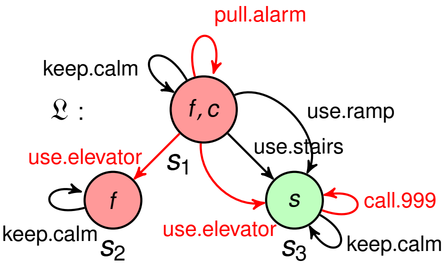

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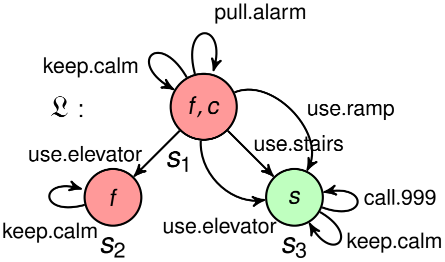

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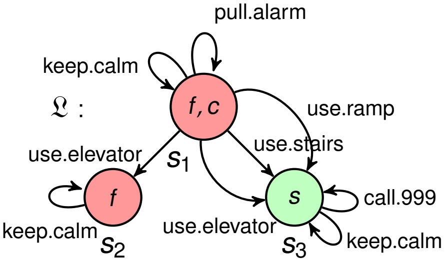

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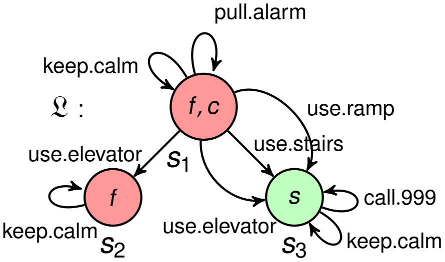

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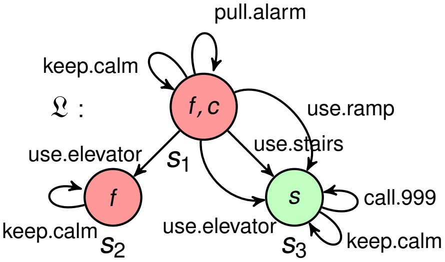

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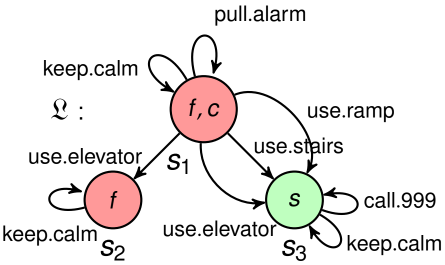

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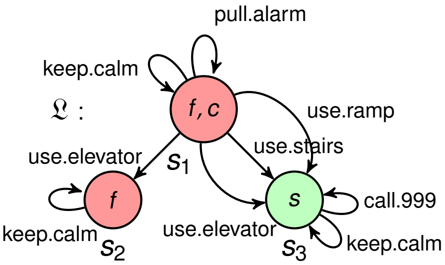

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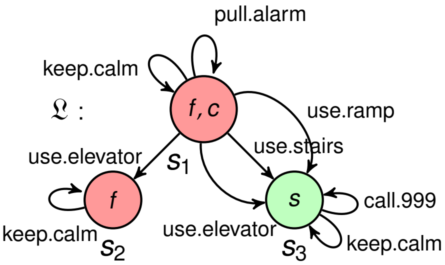

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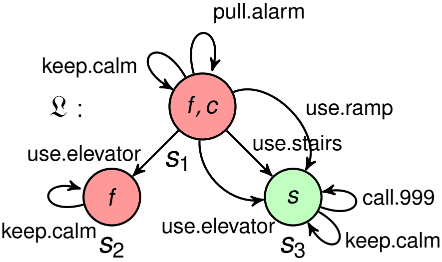

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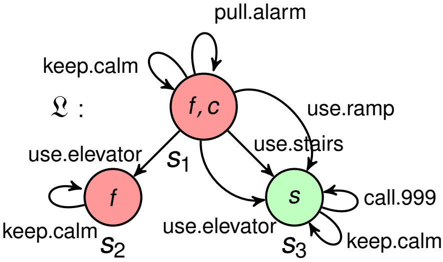

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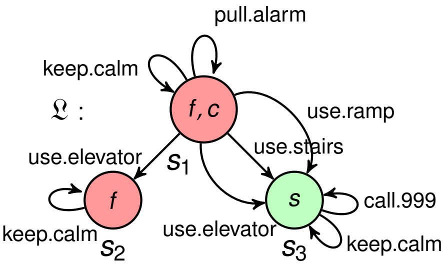

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Axiom system for DLKc

Axioms:	
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